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REVIEWS

Mathematical Structures in Continuous Dynamical Systems. By E. VAN GROESEN & E. M. DE JAGER. North Holland, 1994. 617 pp. ISBN 0 444 821 511. Dfl 295 or US$173.50.

This is really two books in one: the first part by van Groesen is on Poisson structures in fluid dynamics, and the second part by de Jager is a mathematical introduction to the theory of solitons. Nevertheless, the two parts are related in several ways: both deal with partial differential equations (PDEs) from a dynamical systems viewpoint, the PDEs in both parts have Hamiltonian structure or its generalization to Poisson structure, and solitons occur in many approximations to fluid dynamics problems.

The dynamical systems viewpoint of a PDE, when the initial value problem is well posed, is that it describes the time evolution of a point in a function space, for example the space of possible velocity and pressure fields. Implicit in the dynamical systems viewpoint is also a concentration on features which do not depend on the choice of coordinate system for the function space, for example existence of a time-periodic solution rather than an explicit formula for it.

Hamiltonian systems have always formed an important subclass of dynamical systems, because many laws of physics have this special property, beginning with Newton's laws of gravity and including the Euler equations for ideal fluid flow. Why this is so is an interesting philosophical question. One possible answer is that all of them have a variational principle, and it is a simple step from variational principles to Hamiltonian dynamics. But this reply only pushes the question back one stage, and in any case I am not aware that Newton formulated gravitation from a variational principle. Furthermore, it is only ideal fluid systems (no viscosity) that have a Hamiltonian formulation. There is a strong case, however, that this is an important limit and hence it merits study.

Although the first fact one learns about Hamiltonian dynamics is that energy is conserved, there is a much deeper conservation property, discovered by Poincaré, that lies at the heart of what makes Hamiltonian dynamics special, namely preservation of a 'symplectic form'. In its integrated form, for a Hamiltonian system of the standard type $\mathrm{d}p/\mathrm{d}t = -\partial H/\partial q$, $\mathrm{d}q/\mathrm{d}t = \partial H/\partial p$, this says that $\int p \cdot \mathrm{d}q$ round a closed curve in the phase space is preserved under time evolution. One of the consequences of preservation of symplectic form in a Hamiltonian system is that to each generator of a continuous symmetry preserving the symplectic form (if such exist), there is a conserved function. Time evolution is a simple example of such a symmetry, and of course leads to the conservation of energy. But a more profound example is symmetry of a homogeneous fluid under rearrangement. This has no effect on the subsequent evolution. The conservation law to which it leads in the Hamiltonian (that is, inviscid) case is Kelvin's circulation theorem.

Continuous symmetries are the starting point for the generalization from Hamiltonian to Poisson dynamics. Two states can be regarded as equivalent if one can be reached from the other by a symmetry, and then the dynamics induces 'reduced' dynamics on the set of equivalence classes. For example, rearrangement symmetry of a fluid allows one to reduce Newton's laws for the position and velocity of fluid elements to the Euler equations for the velocity field. The reduced dynamics for a Hamiltonian system is, however, only Hamiltonian in a weaker sense. The evolution of any function z on the phase space is given by 'Poisson equations' of the form $dz/dt = \{z, H\}$, where $\{,\}$ is a Poisson bracket, but the bracket is degenerate. For example, the Euler equations for ideal fluid flow are a (degenerate) Poisson system.

Van Groesen guides the reader carefully and clearly through the theory of Poisson systems and develops a wide range of applications to fluid dynamics, from surface waves to coherent vortical structures. A very nice feature is that, acknowledging that the Hamiltonian case is only an idealized limit in fluid mechanics, Part I concludes with a chapter on some effects of dissipation on Poisson systems.

Turning to Part II, solitons, as is now well known, are uniformly travelling spatially localized solutions of certain types of PDE, which survive interaction with each other apart from phase shifts. A famous example is the Korteweg–de Vries (KdV) equation, which was originally derived as a long-wave approximation to surface waves on shallow water.

Mathematically, the underlying feature of soliton-bearing PDEs is that they are 'completely integrable' Hamiltonian systems. For a finite-dimensional Hamiltonian system (of dimension $2n$) this would mean that there are n independent conserved quantities whose Poisson brackets are zero, and it would follow that their levels sets are tori or cylinders on which the motion is conjugate to a straight line flow. For PDEs there is no such clear definition. However, the known soliton PDEs are unified by possession of a 'Lax pair' formulation $dL/dt = [B, L]$, where L and B are linear operators on some function space, depending on the unknown field of the PDE, and $[B, L]$ denotes the commutator $BL - LB$. For the KdV equation, the operator L is simply the Schrödinger operator with the unknown field as potential. The key feature of a Lax pair formulation is that the spectrum of L is conserved, and this plays the role of a complete set of integrals. Then a high point of the theory is that by the technique of inverse scattering it is possible to deduce that any spatially localized initial condition evolves (in both forwards and backwards time) into a superposition of solitons and small amplitude radiation.

There are many clever ideas in the theory of solitons, and de Jager takes the reader on a detailed tour of this remarkable theory, from the Lax pair formulation and inverse scattering to Bäcklund transformations, biHamiltonian structures, and connections with Lie algebras. It is heavy going at times, but a mine of information.

In summary, I feel that this book provides an excellent review of the role of infinite-dimensional Hamiltonian and Poisson dynamical structure in a range of PDE problems. These concepts are finding increasingly many further uses, for example in atmospheric dynamics, magnetohydrodynamics and plasma physics, and merit wider and deeper familiarity.

R. S. MacKay

Topographic Effects in Stratified Flows. By P. G. Baines. Cambridge University Press, 1995. 482 pp. ISBN 0-521-43501-3. £50.

The topographic effects of density stratification of a fluid are an important topic of fluid mechanics because of their intrinsic interest as well as their applications to meteorology and oceanography. Indeed, they play a major role in the momentum budget of the Earth's atmosphere and in the parameterization of general circulation models as well as in the prediction of local weather and currents. Of course, it is the combination of gravity and stratification that produces the buoyancy, which creates the restoring force as well as vorticity, that leads to interesting and important effects. However, the effects of the rotation of the Earth as well as stratification are vital to the larger scales of motion of the atmosphere and oceans, and the effects of rotation have been excluded from the book (as have the effects of condensation and evaporation of water). This exclusion makes possible a more coherent and compact synthesis of research on topographic effects at the cost of a rather artificial division of natural phenomena. Indeed, the author goes on to exclude the effects of compressibility, viscosity and turbulence except for a few sections where they are especially important. But the line has to be drawn somewhere, and his plan is a reasonable one.

There is no comparable book. Since Prandtl's *Essentials of Fluid Dynamics* (1952) many books have devoted a few pages to topographic effects of stratification, and Yih's *Dynamics of Nonhomogeneous Fluids* (1965) and Turner's *Buoyancy Effects in Fluids* (1973) have, each in its way, covered the effects of stratification as a whole, inclusive of topographic effects. But the World Meteorological Organization published two excellent short books: *The Airflow Over Mountains*, edited by Alaka in 1967, and *The Airflow Over Mountains. Research 1958–1972*, edited by Nicholls in 1973. These cover, clearly and comprehensively, the observations and linear theory up to 1972, by which time this research was thoroughly understood, and the latter book covers much of the nonlinear theory. So the gap in the literature which remained to be filled was a comprehensive synthesis of the linear and, especially, nonlinear theory, of numerical results, and of interpretation of laboratory experiments and field observations.

We should be grateful to Professor Baines for filling this gap. He has been working for over 20 years on laboratory experiments and field observations of topographic effects of stratification as well as the theory, and he has evidently taken time and care to write this monograph.

The contents of the book are indicated well by the chapter titles: 1. Background, 2. The flow of a homogeneous layer with a free surface, 3. Two-layer flows, 4. Waves in stratified fluid, 5. Stratified flow over two-dimensional obstacles, 6. Stratified flow past three-dimensional topography, 7. Applications to practical modelling of flow over complex terrain. In some sense chapters 2, 3 and 4 as well as 1 are on background, and so nearly half the book is filled with work leading to understanding the motion of a continuously stratified fluid, but the understanding of this background is an essential pre-requisite. Within this framework various themes are developed: (i) anelastic, Boussinesq, hydrostatic, small- and large-Richardson-number asymptotics, (ii) laminar flow, waves, instability and turbulence, (iii) linear, weakly nonlinear and strongly nonlinear phenomena. The balance seems sensible to me, and it is re-assuring to see that it is recognizably similar to that of the article by Wurtele, Sharman & Datta in the 1996 volume of *Annual Review of Fluid Mechanics*.

As someone who worked on airflow over a mountain in the 1960s and 1970s, I came to this book wondering what is new. My overall impression is that few new fundamental ideas have emerged in the last 20 years, but that a lot of the details, important for practical people, have been filled in by a great increase in numerical simulations, laboratory experiments, and field studies. These are clearly described by word and diagram (there are nearly 200 figures in the book) to give flesh to the theoretical skeleton that came before. There is a great richness in the flows of a stratified fluid past an obstacle, and that richness is revealed.

Analysis of the references cited reveals 47 from the 1990s, 118 from the 1980s, 45 from the 1970s, 41 from the 1960s, 17 from the 1950s and 6 earlier. Of these, 19 were by the author of the book. What does this tell us about the topic, the book and the author? Well, fluid mechanics is a mature subject, and something once discovered endures, so it is still common for us to cite the giants of the nineteenth century although it seems absurd to cite Reynolds whenever we use a Reynolds number or to cite

Newton or Leibniz whenever we use elementary calculus. In particular, the study of airflow over a mountain is about 60 years old, but the author ignores many of the pioneers, notably Lyra (who devised and first applied linear theory) and Dubreil-Jacotin (who discovered Long's model before Long, but did not rival Long's brilliant use of the model in theory and experiment). Again, Professor Baines uses the Taylor–Goldstein equation without either naming it or citing its discoverers. These examples indicate some of the reasons for the distribution of the citations over the decades. However, the implication that a book which only cites recent publications will itself be ephemeral is inapplicable here. The author has chosen to emphasize recent work, so the book's value as a historical source is limited, but the book will have a long shelf life.

There is the usual crop of minor errors and misprints. It was disturbing to note the omission of an important qualification of the very first equation (which is true only if the dynamic viscosity of the fluid is uniform), but this leads to no subsequent error because viscosity is only briefly covered, and then only in the Boussinesq approximation. Such errors are no worse than one expects in any book today, and do little to detract from the overall value of the text.

In all, it is a worthy addition to the series of Cambridge Monographs on Mechanics, which will be of lasting usefulness for research workers on airflow over mountains and on currents over sea mounts.

P. G. DRAZIN